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ABSTRACT

This guidebook on minimum course content presents a study of the nature and methods of proof, using Euclidean geometry as a model. A more rigorous and formal course than usually offered, it is intended for the student who plans to study advanced mathematics. Overall course goals are specified, a course outline is provided, performance objectives are listed, and text references keyed to the performance objectives are provided. A short annotated bibliography is also included. (JP)

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AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

MATHEMATICS: Nature of Proof 5228.33

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DADE COUNTY PUBLIC SCHOOLS
DIVISION OF INSTRUCTION • 1973

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QUINMESTER MATHEMATICS

COURSE OF STUDY
FOR

NATURE OF PROOF

5228.33

(EXPERIMENTAL)

Written by
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for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

A study of the nature and methods of proof, using Euclidean geometry as a model. A more rigorous and formal course than 5218.23. Proofs in Geometry for the student who plans to study advanced mathematics.

Designed for the student who has successfully completed 5228.23 Geometry 3 or excelled in 5218.22 Geometry 2.

TABLE OF CONTENTS

	Page
Goals	3
Introductory Strategies	3
Key to State Adopted Texts	4
Objectives, Scope, Sequence, and Strategies	
I. Introduction to the Nature of Direct Proofs	5
II. Introduction to Indirect Proofs	6
III. Congruent Triangles and Isosceles Triangles	7
IV. Perpendiculars and Parallels	8
V. Quadrilaterals-Properties and Areas	9
VI. Right Triangles-Including Pythagorean Theorem	10
VII. Circles and Spheres	11
Annotated Bibliography	12

OVERALL GOALS

The student will:

1. Use mathematical symbols, notations, and vocabulary pertinent to the study of geometry.
2. Develop his ability to reason formally.
3. Extend his understanding of measurement.
4. Develop reading techniques suitable for mathematics and science.
5. Develop further understanding of geometry.
6. Demonstrate a knowledge of the logical structure of mathematics through the nature of proof.
7. Develop the essentials and techniques of formal proofs in a mathematical system.
8. Develop the techniques of indirect proofs to a mathematical system.

INTRODUCTORY STRATEGIES

Since students have already learned and used the basic properties of geometry, the teacher's objective is to develop the nature of proofs, rather than knowledge of these properties. The properties however, will naturally be strengthened as the student writes proofs.

A minimum of partially completed proofs should be assigned to the students although the teacher shouldn't hesitate to use these as a teaching device at any time they seem to be desirable.

Students will need a commulative list of definitions, postulates, and theorems for reference. This list may be provided by the teacher or recorded by the student.

Although most proofs will be formal, deductive proofs, the teacher should utilize indirect proofs frequently.

This quin is most flexible because the purpose is <u>nature</u>, <u>not content</u>.

KEY TO STATE ADOPTED TEXTS

M - Moise and Downs. Geometry. Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1967.

L - Lewis, Harry. Geometry, A Contemporary Course. Princeton, New Jersey: D. Van Nostrand Co., Inc., 1968.

J - Jurgensen, Donnelly, Dolcini. Modern Geometry. Boston, Massachusetts: Houghton Mifflin Co., 1965.

A - Anderson, Garon, Gremillion. School Mathematics Geometry. Boston, Massachusetts: Houghton Mifflin Co., 1966.

I. INTRODUCTION TO THE NATURE OF DIRECT PROOFS

Performance Objectives

The student will:

1. Learn to work within a "structured" discipline where he must disregard previously accepted concepts.
2. Be able to separate statements into Hypothesis-Conclusion form.
3. Understand how a deductive system works.

Course Content

Difference between undefined terms, definitions, postulates and theorems.

Non-formal proofs
- Point-Plotting Theorem
- Mid-point Theorem
- Intersection of two lines
- Line intersecting a plane
- Determining a plane

Introduction to formal proofs
- Use of two column style to prove theorems involving complementary, supplementary, and vertical angles.

Suggested Strategies

1. Since students have used the properties of Geometry in previous quins, some will challenge the need to 'prove' them. It is important that they develop the understanding and appreciation of a deductive mathematical system.
2. Students will need a notebook in which they list the definitions, postulates, and theorems as they are introducted. Students are restricted to using only these.
3. Do not hesitate to use models.
4. Try to develop the attitude; "I know it's true, but can I prove it"?

State Adopted References

	M	L	J	A
Ch.	1-4	1,7	1-4	1-4

II. INTRODUCTION TO INDIRECT PROOFS

Performance Objectives

The student will:

1. Understand how to reason inductively.
2. Prove wanted concepts by showing contradictory possibilities and the elimination of the inconsistent conditions.

Course Content

Induction

Paragraph form

Parallelism

Introduction to indirect proofs

Suggested Strategies

1. It is important to develop the proofs of the properties which the students have previously studied. A large majority can be proved with more ease by the inductive and indirect proof approach.
2. Notebooks should be continued.
3. Examples and models are very helpful.
4. Developing the method of elimination to find their conclusion will help lead students to the proof.
5. Most important in developing indirect proofs is to understand the Laws of the Excluded Middle and Contration.
6. Partially constructed proofs will help in the beginning.
7. The Paragraph form is easier for indirect proofs rather than the two-column form used in formal proofs.

State Adopted References

	M	L	J	A
Ch.		8,9	5	

III. CONGRUENT TRIANGLES AND ISOSCELES TRIANGLES

Performance Objectives

The student will:

1. Use the congruence postulates in formal proofs to prove that triangles are congruent or that corresponding parts are congruent.
2. Use the postulates to further prove special properties of isosceles and equilateral triangles.
3. Solve proofs involving overlapping triangles.

Course Content

Congruence Postulates
SAS, ASA, SSS

Isosceles and Equilateral Triangles

Overlapping Triangles

Angle Bisectors

State Adopted References

	M	L	J	A
Ch.	5	5,15	6	5-7

Suggested Strategies

1. At this time have students include <u>all</u> steps in the proof. Omission of "obvious" steps can be made later, but the emphasis in this unit is on assuming <u>nothing</u>.

IV. PERPENDICULARS AND PARALLELS

Performance Objectives	Course Content
The student will:	Perpendiculars
1. Write indirect proofs by making a supposition and reaching a contradiction.	Parallels
2. Use indirect proofs to prove properties of perpendicularity and parallelism.	Indirect proofs
3. Recognize a characterization.	Characterizations
4. Use auxiliary sets in writing proofs.	Auxiliary Sets
5. Write direct formal proofs of perpendicularity and parallelism.	

State Adopted References

	M	L	J	A
Ch.	6,8, 9,10, 15	2,4, 6-9	4,5	9-11

Suggested Strategies

1. Partially constructed indirect proofs will be necessary until students are familiar with the technique.

V. QUADRILATERALS-PROPERTIES AND AREAS

Performance Objectives

The student will:

1. Prove the properties of a parallelogram, rhombus, rectangle, and square.
2. Prove that a given quadrilateral is a parallelogram, rhombus, rectangle, or square.
3. Develop the formulas for the areas of a rectangle, parallelogram, triangle, trapezoid, and other special polygons.

Course Content

Polygons

Polygonal Regions

Special Polygons
- Quadrilaterals
- Trapezoid
- Parallelogram
- Rhombus
- Rectangle
- Square
- Kite

State Adopted References

	M	L	J	A
Ch.	9,11	2,4 6-9	6-13	10,13

Suggested Strategies

1. In developing the special properties of a parallelogram, take the opportunity to point out the "building block" structure.

VI. RIGHT TRIANGLES

Performance Objectives

The student will:

1. Prove the median to the hypotenuse is one-half the measure of the hypotenuse. (And converse)
2. Prove the Pythagorean Theorem. (And converse)
3. Prove special properties of a 30-60-90 triangle.
4. Prove the special properties of an isosceles right triangle.
5. Use the Pythagorean Theorem to prove the Distance Formula.
6. Use the coordinate system to prove certain properties of right triangles and quadrilaterals.

Course Content

Special properties of right triangles
- Median to hypotenuse
- Altitude to hypotenuse

Special right triangles
- 30-60-90
- Isosceles

Pythagorean Theorem

Introduction to coordinate geometric proofs

State Adopted References

	M	L	J	A
Ch.	9,11, 12, 13	11	7,11 12	13

Suggested Strategies

1. Introduce as many versions of proofs of the Pythagorean Theorem as possible. This will help student appreciation of the nature of mathematics.
2. When using coordinate geometry, have students use slope, midpoint, and other concepts without proof. Place emphasis on the use of the Pythagorean Theorem applications. Do not dwell on coordinate geometry since this follows in detail in a later quin.

VII. CIRCLES AND SPHERES

Performance Objectives

The student will:

1. Prove statements about the relationship of a tangent and radius of a given circle.
2. Prove properties of relationship between chords and perpendiculars.
3. Prove properties of inscribed angles and intercepted arcs.
4. Prove other properties of relationships between parts of a circle. (See course content.)
5. Prove the value of pi.

Course Content

Relationship of radius and tangent

Relationship of chord and perpendicular from center of circle

Inscribed angles and intercepted arcs

Central angles

Secants

Power of a point

Inscribed circles

Circumscribed circles

Pi

Area of circle and sections

Lengths of arcs

State Adopted References

	M	L	J	A
CH.	14,16	12	9,14	16,18

Suggested Strategies

1. Any of the course content that applies to spheres should be utilized, if time permits.
2. Models are especially helpful when teaching spheres.
3. When talking about power of a point, try to have students understand what is meant by the "power" of that point.
4. The history of pi is fascinating but can't be given too much time. Try to develop a mathematical concept of pi.
5. Students should note how triangles are utilized in working with circles.

ANNOTATED BIBLIOGRAPHY

References: Not State Adopted

Fundamental Concepts of Geometry, Meserve. Addison-Wesley Publishing Co., Inc. Ready, Mass. 1959.

Excellent source of in-depth material. Good discussion of the various types of proof.

Geometry and Its Methods, Fujii. John Wesley and Sons, Inc. New York, New York. 1969.

Easy reading. Could be good for student reference.

Modern Geometry, Nichols, et al. Holt, Rinehart, and Winston, Inc., New York, New York. 1968.

Another textbook approach.

Exploring Geometry, Keedy, et al. Holt, Rinehart, and Winston, Inc., New York, New York. 1967.

Textbook approach with some "applied" problems.

Post Test

It is not felt that a single test can measure a student's ability to handle proofs. However, a "final" test should measure a student's ability to do both formal, deductive proofs and indirect, inductive proofs.